Math 70 : 3.6 & 3.7 Slope-intercept and Point-Slope forms

Lesson 7

## 3.6 Slope-Intercept Form of the equation of a line

Objectives

1) Graph a line from slope and y-intercept
2) Determine if two lines are parallel, perpendicular, or neither using equations.

ZOOM SQUARE

## 3.7 Point-slope Formula for Equation of a Line

Objectives

1) Write an equation of a line given
   - slope and one point (ordered pair)
   - two ordered pairs
2) Graph a line given slope and a point that's not the y-intercept.

* This section also includes linear regression which we will skip completely. *

Graphing linear equations
GC 19 Windows for Applications
GC 20 Graphing More than one function
Writing Linear Equations.

Note: Both of these sections are review of Math 45!

**Math 60 Summary of Techniques for Graphing a Line**

Begin with Option 1. If it does not apply, try Option 2. If not Option 3, go on to Option 4, and so on.

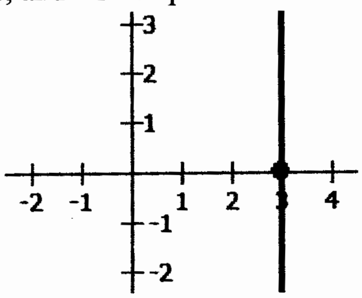
Option 1:

Ask: Is the line vertical? [Does the equation have x but no y?]

Method: Plot x-intercept and a line up and down from it.

Example 1: $x = 3$.

Plot the x-intercept at the value given, (3,0) in this example, and a line up and down from it.




Graph for Example 1

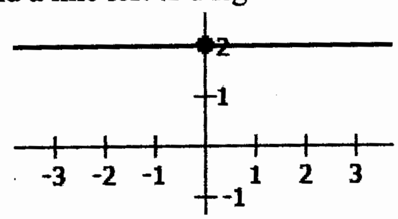
Option 2:

Ask: Is the line horizontal? [Does the equation have y but no x?]

Method: Plot the y-intercept and a line left and right from it.

Example 2: $f(x) = 2$

Plot the y-intercept at the value given, (0,2) in example, and a line left and right from it.




Graph for Example 2

Option 3:

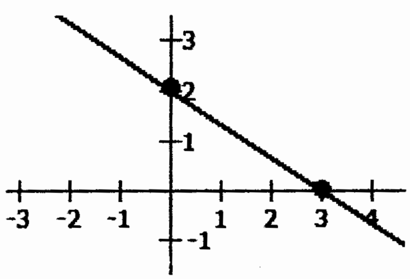
Ask: Are the x-intercept and y-intercept both integers? [Is the constant is evenly divisible by both the coefficient of the x-term and evenly divisible by the coefficient of the y-term?]

Method: Find and plot the x-intercept, find and plot the y-intercept, connect the two with a line.

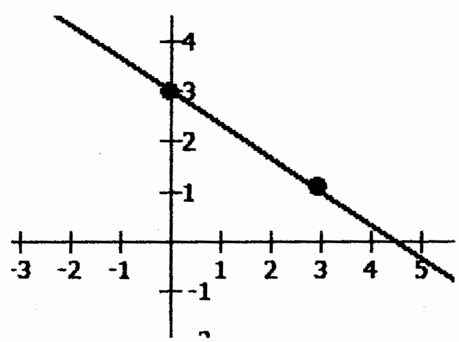
Example 3: $2x + 3y = 6$ [6 is divisible by 2 and divisible by 3]

Find the x-intercept (set y=0, solve for x), (3,0) in example, and plot it.

Find the y-intercept (set x=0, solve for y), (0,2) in example, and plot it. Connect with a line.




Graph for Example 3




Graph for Example 4

Option 4:

Ask: Is the y-intercept an integer? [Is the constant term is divisible by the y-coefficient?]

Method: Write equation in slope-intercept ($f(x) = mx + b$) form, plot the y-intercept, use the slope.

Example 4: $2x + 3y = 9$ [9 is divisible by y-coefficient 3, but not by x-coefficient 2]

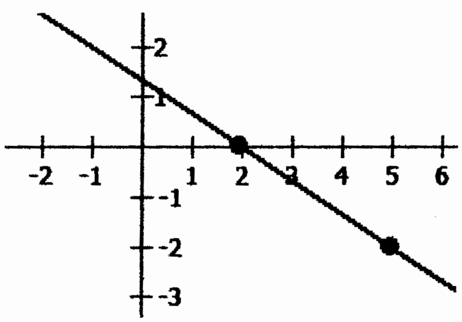
Write in slope-intercept form: $f(x) = -\frac{2}{3}x + 3$.

Continued…

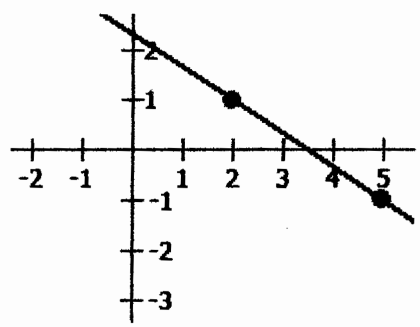
Plot the y-intercept, (0,3) in example. (continued on the back)
Write slope as $\frac{rise}{run}$. ($\frac{-2}{3}$ in example).
From the y-intercept go up *rise* units (if *rise* is positive) or down *rise* units (if *rise* is negative).
From there, go right *run* units (if *run* is positive) or left *run* units (if *run* is negative).

Option 5:
Ask: Is the x-intercept an integer? [Is the constant term is divisible by the x-coefficient?]
Method: Write equation in slope-intercept ($y = mx + b$) form, find and plot the x-intercept, use the slope.

Example 5: $2x + 3y = 4$ [4 is divisible by x-coefficient 2 but not by y-coefficient 3]
Write in slope-intercept form: $f(x) = -\frac{2}{3}x + \frac{4}{3}$
Find and plot x-intercept (2,0).
Write slope as $\frac{rise}{run}$. ($\frac{-2}{3}$ in example)
From the x-intercept go up *rise* units (if *rise* is positive) or down *rise* units (if *rise* is negative).
From there, go right *run* units (if *run* is positive) or left *run* units (if *run* is negative).



Graph for Example 5



Graph for Example 6

Option 6:
Ask: Is neither the x-intercept nor y-intercept an integer? [Is the constant term is not divisible by either the x-coefficient or the y-coefficient?]
Method: Find any point and use the slope.

Example 6: $2x + 3y = 7$. [7 is not divisible by 2 or by 3]
Choose an x-value, substitute, and solve for y, OR choose a y-value, substitute, and solve for x.]
Choosing x=0 or x=1 in this example give fractions for y. Choose x=2.

$2(2) + 3y = 7 \qquad 4 + 3y = 7 \qquad 3y = 3 \qquad y = 1$

Plot the point ( in this example, (2,1) )
Write the equation in slope-intercept form. ($y = -\frac{2}{3}x + \frac{7}{3}$ in this example)
Write slope as $\frac{rise}{run}$. ($\frac{-2}{3}$ in example)
From the point go up *rise* units (if *rise* is positive) or down *rise* units (if *rise* is negative).
From there, go right *run* units (if *run* is positive) or left *run* units (if *run* is negative).

**Math 60 How to Write the Equation of a Line as a Linear Function**

Step 1: Recognize if the line is vertical. Write equation $x = x - coordinate$.

How to know if a line is vertical:

- It says "vertical".
- It says "slope undefined".
- It is parallel to another line with undefined slope (vertical).
- It is parallel to another line whose equation is $x = x - coordinate$ (vertical).
- It is perpendicular to a horizontal line, $y = y - coordinate$.
- It is perpendicular to a horizontal line, slope = 0.
- It is parallel to the y-axis.
- It is perpendicular to the x-axis.

IMPORTANT: Vertical lines are not functions!

Step 2: Recognize if the line is horizontal. Write equation $f(x) = y - coordinate$.

How to know if a line is horizontal?

- It says "horizontal".
- It says "slope 0".
- It is parallel to another line with zero slope (horizontal).
- It is parallel to another line whose equation is $y = y - coordinate$ (horizontal).
- It is perpendicular to a vertical line, $x = x - coordinate$.
- It is perpendicular to a vertical line, slope undefined.
- It is parallel to the x-axis.
- It is perpendicular to the y-axis.

Step 3: Given slope and a point:

If the point is the y-intercept $(0, b)$, substitute into $f(x) = mx + b$.

If the point is not the y-intercept, substitute into the point-slope formula $y - y_1 = m(x - x_1)$, simplify, and replace $y$ by $f(x)$.

Step 4: Given two points:

Find the slope using the slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

If the point is not the y-intercept, substitute into the point-slope formula $y - y_1 = m(x - x_1)$, simplify, and replace $y$ by $f(x)$.

Step 5: Given "parallel to _____" and a point.

Find the slope of the given line by writing in $y = mx + b$

Use that same slope.

If the point is not the y-intercept, substitute into the point-slope formula $y - y_1 = m(x - x_1)$, simplify, and replace $y$ by $f(x)$.

Step 6: Given "perpendicular to _____" and a point.

Find the slope of the given line by writing in $y = mx + b$

Take the opposite and reciprocal of that slope to get the new slope.

If the point is not the y-intercept, substitute into the point-slope formula $y - y_1 = m(x - x_1)$, simplify, and replace $y$ by $f(x)$.

**Math 70: 3.6 & 3.7** **Slope-Intercept Form of the Equation of a Line**
**Point-Slope Formula for the Equation of a Line**

**Examples and Practice**

1) Write the equation of a line having slope $\frac{2}{3}$ and y-intercept $\left(0,-\frac{5}{4}\right)$

2) Write the equation of a line with slope $\frac{1}{5}$ passing through the point $(4,-6)$

3) Write the equation of a line passing through $\left(\frac{1}{2},\frac{1}{4}\right)$ and $\left(\frac{3}{2},\frac{3}{4}\right)$ in function notation.

4) Write the equation of lines
   a. Slope undefined, through $(-1,3)$
   b. Slope 0, through $(-2,-5)$

5) Find the slope of a line perpendicular to $2x-3y=1$.

6) Write equation of a line perpendicular to $2x-3y=1$ through $(-4,8)$ in function notation.

7) Find the slope of a line parallel to $2x-3y=1$.

8) Write a linear function parallel to $2x-3y=1$ passing through $(-4,8)$

9) The number of McDonald's restaurants in 2010 was 32,737. In 2005, there were 31,046. Let y be the number of McDonald's restaurants and x be the number of years since 2005.
   a. Write a linear equation that models the growth of McDonald's.
   b. Use equation to predict number of McDonald's restaurants in 2013.

10) A company has learned that by pricing a swim noodle at $3, sales will be 10000 noodles per day, while raising the price to $5 causes sales to drop to 8000 noodles per day. Let x be price and y be number of noodles sold.
   a. Assume the relationship between price and number sold is linear and write the equation.
   b. Predict the number of noodles sold if the price is $3.50.

① Write equation of a line having slope $\frac{2}{3}$ and y-int $(0, -\frac{5}{4})$.

Equation of a line in <u>slope-intercept form</u>

$y = mx + b$

$(x, y)$ any ordered pair (point) on the line = variables in eqn

$m$ = slope

$b$ = y-coord of yint $(0, b)$

* Memorize

Equation has $(x, y)$ variables.

Replace $m$ by slope $\frac{2}{3}$

Replace $b$ by yint $-\frac{5}{4}$.

$$\boxed{y = \frac{2}{3}x - \frac{5}{4}}$$

check $(0, -\frac{5}{4})$
$(x, y)$

$$-\frac{5}{4} = \frac{2}{3}(0) - \frac{5}{4}$$

$$-\frac{5}{4} = 0 - \frac{5}{4} \checkmark$$

② Write equation of a line with slope $\frac{1}{5}$ through $(4, -6)$

<u>Notice</u>: $(4, -6)$ is <u>not</u> on the y-axis, so it's <u>not</u> the y-int and cannot be substituted for b.

<u>Method 1</u>: Subst $m = \frac{1}{5}$ and $(x, y) = (4, -6)$ } step 1 in $y = mx + b$.

$$y = mx + b$$

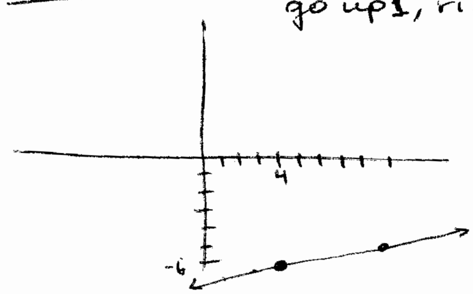
$$-6 = \frac{1}{5}(4) + b$$

$$-6 = \frac{4}{5} + b$$

$$-6 - \frac{4}{5} = b$$

$$-\frac{34}{5} = b$$

<u>Bonus</u>: Graph: plot $(4, -6)$ go up 1, right 5




Rewrite eqn w/ b and m } step 2

$$\boxed{y = \frac{1}{5}x - \frac{34}{5}}$$

another method – next page

Method 2: Subst into $y - y_1 = m(x - x_1)$

**Point-slope formula for equation of a line**

$$y - y_1 = m(x - x_1)$$

$(x, y)$ any ordered pair on the line = variables in eqn.

$m$ = slope

$(x_1, y_1)$ = a specific point (given) on the line

$y - (-6) = \frac{1}{5}(x - 4)$ — Step 1: subst $m = \frac{1}{5}$, $(x_1, y_1) = (4, -6)$

$y + 6 = \frac{1}{5}x - \frac{4}{5}$ — Step 2: simplify by dist

$y = \frac{1}{5}x - \frac{4}{5} - 6$ — Step 3: isolate $y$

$$\boxed{y = \frac{1}{5}x - \frac{34}{5}}$$

Notice: You get the same answer regardless of the method.

③ Write the equation of a line through $\left(\frac{1}{2}, \frac{1}{4}\right)$ and $\left(\frac{3}{2}, \frac{3}{4}\right)$, in function notation.

* Method 1 $y = mx + b$ OR Method 2 $y - y_1 = m(x - x_1)$

BOTH REQUIRE $m = \frac{y_2 - y_1}{x_2 - x_1}$ first

Step 1: find slope

$$m = \frac{\frac{3}{4} - \frac{1}{4}}{\frac{3}{2} - \frac{1}{2}}$$

$$m = \frac{\frac{2}{4}}{\frac{2}{2}}$$

$$m = \frac{1}{2}$$

Method 1 $y = mx + b$

* It doesn't matter whether $(x_1, y_1) = (\frac{1}{2}, \frac{1}{4})$ or $(x_1, y_1) = (\frac{3}{2}, \frac{3}{4})$ } use only one

step 2: subst

$\frac{1}{4} = \frac{1}{2}(\frac{1}{2}) + b$ OR $\frac{3}{4} = \frac{1}{2}(\frac{3}{2}) + b$

step 3: isolate b

$\frac{1}{4} = \frac{1}{4} + b$ $\frac{3}{4} = \frac{3}{4} + b$

$0 = b$ $0 = b$

step 4 rewrite

$$y = \frac{1}{2}x$$

"Write in function notation" means use f(x) in place of y

$$\boxed{f(x) = \frac{1}{2}x}$$

Method 2 $y - y_1 = m(x - x_1)$

step 2: Write the point-slope formula.

$$y - y_1 = m(x - x_1)$$

step 3: Substitute $m = \frac{1}{2}$, $x_1 = \frac{1}{2}$, $y_1 = \frac{1}{4}$ (or $x_1 = \frac{3}{2}$ and $y_1 = \frac{3}{4}$ if you prefer).

$$y - \frac{1}{4} = \frac{1}{2}(x - \frac{1}{2})$$

step 4: Simplify to $y = mx + b$ form

$$y = \frac{1}{2}x - \frac{1}{4} + \frac{1}{4}$$

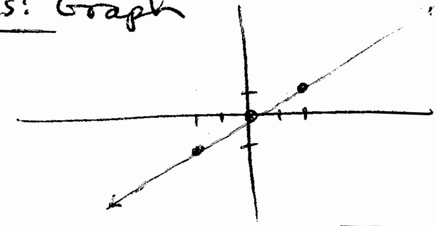
$$y = \frac{1}{2}x$$

step 5: Replace y by f(x) to get function notation

$$\boxed{f(x) = \frac{1}{2}x}$$

y int (0,0)
slope $\frac{1}{2}$

Bonus: Graph



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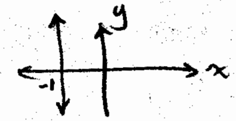
(4) Write equations of lines

a) slope undefined, through $(-1, 3)$

step 1: Undefined slope means vertical.
Vertical means $x = x$-coordinate

step 2: $\boxed{x = -1}$

Graph:

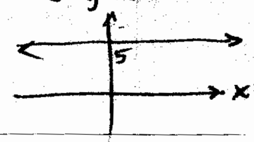



b) slope 0, through $(-2, -5)$

step 1: Slope 0 means horizontal.
Horizontal means $y = y$-coordinate

step 2: $\boxed{y = -5}$

Graph:

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(5) Find the slope of a line perpendicular to $2x - 3y = 1$

step 1: Find slope of given line $2x - 3y = 1$ by solving for $y$.

$$2x - 3y = 1$$

$$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{1}{-3}$$

$$y = \frac{2}{3}x - \frac{1}{3}$$

$$\text{slope} = \frac{2}{3}$$

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step 2: Determine slope of new line.
Perpendicular means opposite & reciprocal slope

new slope $= -\frac{3}{2}$

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⑥ Write equation of a line perpendicular to $2x - 3y = 1$ through $(-4, 8)$. in function notation

* Use the work from ⑤ *

Step 3: Write the point-slope formula and substitute the new slope and given point.

$$y - y_1 = m(x - x_1)$$

$$y - 8 = -\frac{3}{2}(x + 4)$$

$$y = -\frac{3}{2}x - 6 + 8$$

$$\boxed{f(x) = -\frac{3}{2}x + 2}$$

OR $y = mx + b$ method

Subst $y = 8$
$x = -4$
$m = -\frac{3}{2}$

$$8 = -\frac{3}{2}(-4) + b$$

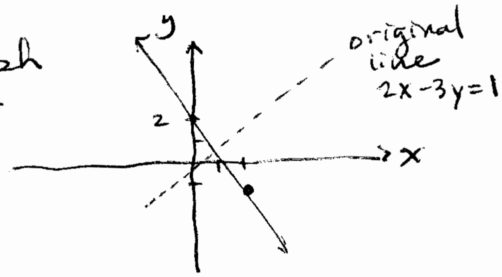
$$8 = 6 + b$$

$$2 = b$$

$$y = -\frac{3}{2}x + 2$$

$$\boxed{f(x) = -\frac{3}{2}x + 2}$$

Graph
y-int 2
slope $-\frac{3}{2}$
down 3, right 2




⑦ Find the slope of a line parallel to $2x - 3y = 1$.

Step 1: Find the slope of $2x - 3y = 1$ (same as ⑤!)

$$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{1}{-3}$$

$$y = \frac{2}{3}x - \frac{1}{3}$$

$$m = \frac{2}{3}$$

Step 2: A line parallel has the same slope.

$$\boxed{m = \frac{2}{3}}$$

(8) Write a linear function parallel to $2x - 3y = 1$ passing through $(-4, 8)$.

<u>step 1</u>: same as step 1 in previous example. $m = \frac{2}{3}$

<u>step 2</u>: slope of line parallel means same slope.

new slope $= \frac{2}{3}$

<u>step 3</u>: Substitute & simplify.

$$y - 8 = \frac{2}{3}(x + 4)$$

$$y = \frac{2}{3}x + \frac{8}{3} + 8$$

$$\boxed{f(x) = \frac{2}{3}x + \frac{32}{3}}$$

M70

(9) The number of McDonald's restaurants in 2010 was 32,737. In 2005, there were 31,046. Let y = # McDonald's restaurants and x = # years since 2005.

a) Write a linear equation that models the growth of McDonald's.

b) Use equation to predict number of McDonald's restaurants in 2013.

Step 1: Identify ordered pairs.

(2010, 32737)

(2005, 31046)

Step 2: Rewrite years as # years since 2005.

$$\begin{array}{r} 2010 \\ -\,2005 \\ \hline 5 \end{array} \Rightarrow (5, 32737)$$

$$\begin{array}{r} 2005 \\ -\,2005 \\ \hline 0 \end{array} \Rightarrow (0, 31046).$$

Step 3: Find slope using slope formula

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{32737 - 31046}{5 - 0} = \frac{1691}{5}$$

Step 4: Notice y-intercept is given! Use short method.

$$y = mx + b$$

$$\boxed{y = \frac{1691}{5}x + 31046}$$ (from (0, 31046)!)

OR $$\boxed{y = 338.2x + 31046}$$

Step 5: Evaluate for 2013 and round to nearest whole.

$$\begin{array}{r} 2013 \\ -\,2005 \\ \hline 8 = x \end{array}$$

$$y = 338.2(8) + 31046$$

$$= 33751.6 \approx \boxed{33752 \text{ restaurants}}$$

(10) A company prices a swim noodle at \$3 and sells 10000 per day, but at \$5 they sell only 8000 per day.
Let $x$ = price and $y$ be # noodles sold.
a) Assume linear, write linear equation.

step 1: Identify ordered pairs given
$(x, y) = (\text{price}, \#\text{sold}) \Rightarrow (3, 10000)$ and $(5, 8000)$

step 2: find slope of line
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10000 - 8000}{3 - 5} = \frac{2000}{-2} = -1000$$

m is negative?! What does that mean?
- If you graph line, it goes downhill
- The sales of noodles decrease as price goes up.

step 3: Use point slope formula
$$y - y_1 = m(x - x_1)$$
$$y - 8000 = -1000(x - 5)$$ subst m and either point
$$y - 8000 = -1000x + 5000$$ dist
$$\boxed{y = -1000x + 13000}$$ collect constants

b) Predict # noodles if price is \$3⁵⁰.
subst $x = 3.5$
$$y = -1000(3.5) + 13000$$
$$= -3500 + 13000$$
$$= \boxed{9500 \text{ noodles}}$$

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Extra Practice with Parallel, Perpendicular, Neither

Determine if the given lines are parallel, perpendicular or neither.

① $3x + 7y = 21$ (A)

$6x + 14y = 7$ (B)

Step 1: write each equation in $y = mx + b$ to find slopes

(A) $3x + 7y = 21$ isolate $y$

$-3x \quad\quad -3x$

$\frac{7y}{7} = \frac{-3x}{7} + \frac{21}{7}$

$y = -\frac{3}{7}x + 3 \implies m_A = -\frac{3}{7}$

(B) $6x + 14y = 7$

$-6x \quad\quad -6x$

$\frac{14y}{14} = \frac{-6x}{14} + \frac{7}{14}$

$y = \frac{-3}{7}x + \frac{1}{2} \implies m_B = \frac{-3}{7}$

Step 2: If the two slopes are the same the lines are parallel

$m_A = -\frac{3}{7}$ and $m_B = \frac{-3}{7}$

**parallel**

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② $-x + 3y = 2$ (A)

$2x + 6y = 5$ (B)

step 1: write each equation in $y = mx+b$ to find slopes.

isolate y

(A) $-x + 3y = 2$

$\quad +x \qquad +x$

$\frac{3y}{3} = \frac{x}{3} + \frac{2}{3}$

$y = \frac{1}{3}x + \frac{2}{3}$ $\qquad m_A = \frac{1}{3}$

(B) $2x + 6y = 5$

$\quad -2x \qquad -2x$

$\frac{6y}{6} = \frac{-2x}{6} + \frac{5}{6}$

$y = \frac{-1}{3}x + \frac{5}{6}$ $\qquad m_B = \frac{-1}{3}$

step 2: If slopes are opposites ($\frac{1}{3}$ and $-\frac{1}{3}$) but NOT reciprocals, they are not the same (so not parallel) and not opposite reciprocals (so not perpendicular).

**Neither**

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③ $2x - 3y = 12$ (A)

$6x + 4y = 16$ (B)

Step 1 write each equation in $y = mx + b$ to find slopes.

(A) $2x - 3y = 12$ isolate $y$

$$\begin{aligned} 2x - 3y &= 12 \\ -2x \quad & \quad -2x \end{aligned}$$

$$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{12}{-3}$$

$$y = \frac{2}{3}x - 4 \qquad m_A = \frac{2}{3}$$

(B) $6x + 4y = 16$

$$\begin{aligned} 6x + 4y &= 16 \\ -6x \quad & \quad -6x \end{aligned}$$

$$\frac{4y}{4} = \frac{-6x}{4} + \frac{16}{4}$$

$$y = -\frac{3}{2}x + 4 \qquad m_B = -\frac{3}{2}$$

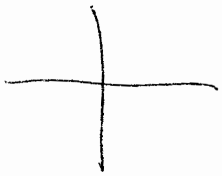
Step 2 If slopes are both opposites and reciprocals, the lines are perpendicular

**perpendicular**

(4) $x = 2$

$y = -10$

Recognize that $x = 2$ is vertical and $y = -10$ is horizontal.



which makes these lines

**perpendicular**.

CAUTION: With vertical and horizontal lines, do not try to take the reciprocal of 0 or undefined slope! Imagine the graphs.

(5) $x = 3$

$x = -10$

both vertical $\Rightarrow$ **parallel**

(6) $y = -4$

$y = 7$

both horizontal $\Rightarrow$ **parallel**

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(7) $x = 3$ (A)

$2x - 3y = 5$ (B)

(A) $x = 3$ is vertical. $m_A =$ undefined

(B) $2x - 3y = 5$

$$-2x \qquad -2x$$

$$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{5}{-3}$$

$$y = \frac{2}{3}x - \frac{5}{3} \qquad m_B = \frac{2}{3}$$

**neither**

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(8) The product of the slopes is $-1$.

$m_A \cdot m_B = -1$ product of slopes

$m_A = \frac{-1}{m_B}$ isolate $m_A$

opposite reciprocals $\Rightarrow$ **perpendicular**

(9) The slopes are the same and the y-intercepts are the same.

$y = m_1 x + b_1$
$y = m_1 x + b_1$ } They are the same line. Only one line

**neither**

(10) The slopes are the same and the y-intercepts are different

$y = m_1 x + b_1$
$y = m_1 x + b_2$ } different y-ints $\Rightarrow$ different lines with same slopes

**parallel**

(11) The product of the slopes is 2.

$m_1 \cdot m_2 = 2$

$m_1 = \frac{2}{m_2}$

$\sqrt{2} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$

not the same slope / unless $m_1 = m_2 = \sqrt{2}$ parallel

not opposite reciprocals

**neither**

(12) The product of the slopes is 0.

$m_1 \cdot m_2 = 0$ means $m_1 = 0$ or $m_2 = 0$ or both.

**if both $m_1 = m_2 = 0$ then parallel otherwise neither**

Name ______________________________

Date ______________________________

## TI-84+ GC 19 Choosing an Appropriate Window for Applications

Objective: Choose appropriate window for applications

**Example 1:** A small company makes a toy. The price of one toy $x$ (in dollars) is related to the number of toys which are sold $y$ by the equation $y = -250x + 3500$. What types of numbers do not make sense for $x$? What types of numbers do not make sense for $y$? Graph $y = -250x + 3500$ in an appropriate window.

Step 1: What values make sense? Which maximum and minimum values are easiest to calculate?

$y$, the number of toys, must be a whole number—not a decimal nor a negative. $x$, the price, must be positive. Since $x$ is in dollars, it should be rounded to two decimal places, unless the instructions say "nearest dollar".

| We need Xmin, Xmax, Ymin, and Ymax. From any two, we can find the rest. |
|---|

While it seems silly to give the toy away (price $x$ = \$0), this is easy to find. Xmin = 0.
The smallest number of toys that might be sold is none. Ymin=0.

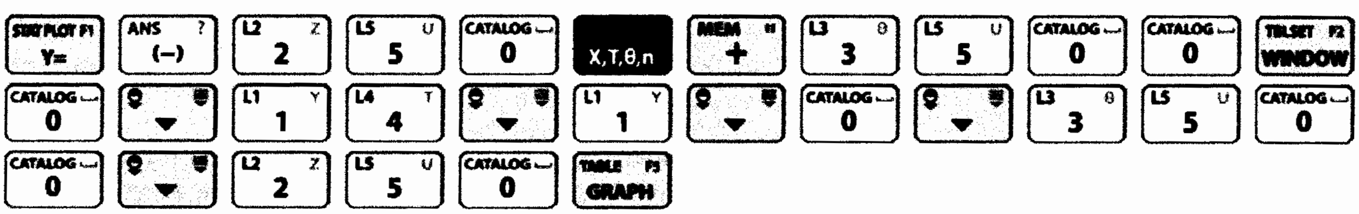
| CAUTION: Ymin may correspond to Xmax, not Xmin! |
|---|

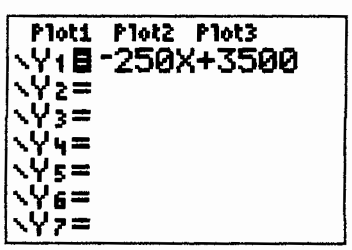
Step 2: Substitute to find ordered pairs on the graph to find remaining upper and lower values.
Find the value of $y$ which corresponds to Xmin = 0. $y = -250(0) + 3500 = 3500$ Ymax = 3500
When $x = 0$, $y = 3500$ means they'll give away 3500 toys at a price of \$0. (0,3500) is a point on the graph.

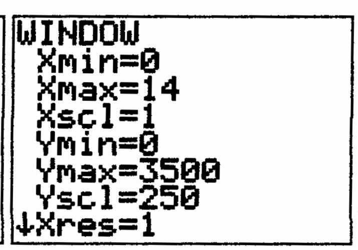
Find the value of $x$ which corresponds to Ymin = 0. $0 = -250x + 3500$ gives Xmax = 14.
When $x = 14$, $y = 0$ means they'll sell no toys if the price is \$14. (14,0) is a point on the graph.

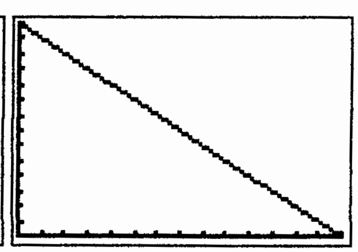
Step 3: Determine Xscl and Yscl.
Xmax – Xmin = 14 – 0 = 14. 14 is divisible by 2 (7 ticks) or 1 (14 ticks). Xscl = 1.
Ymax – Ymin = 3500 – 0 = 3500. 3500 is divisible by 500 (7 ticks) or 250 (14 ticks). Yscl = 250.

Step 4: Graph.



**Example 2:** The equation $y(x) = 0.212x + 12.04$ tells the percentage of the labor force $y$ which are adults over age 65 for $x$ the number of years since 1990. What types of numbers do not make sense for $x$? What types of numbers do not make sense for $y$? Graph $y(x) = 0.212x + 12.04$ in an appropriate window. Interpret the graph.

<u>Step 1:</u> What values make sense? Which maximum and minimum values are easiest to calculate?

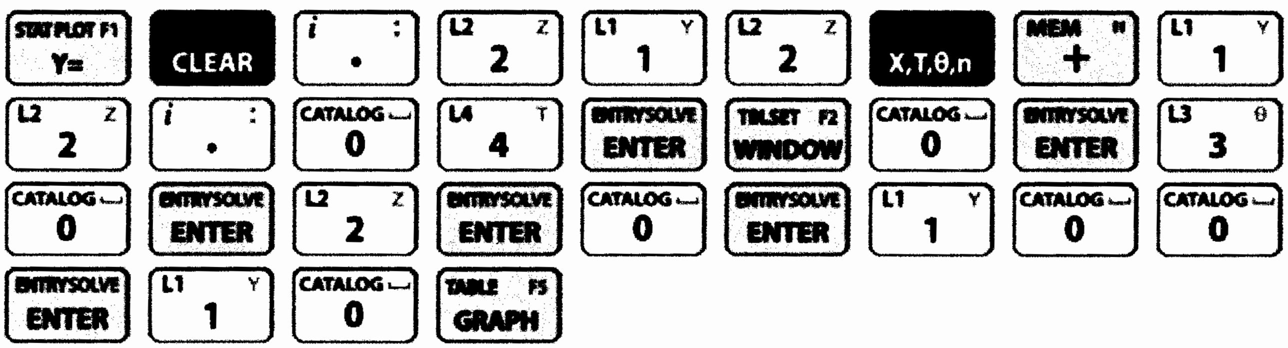
$x$ the number of years, is a whole number. If we consider 1990 and 2020, we get $x = 0$ and $x = 30$, giving Xmin = 0 and Xmax = 30.

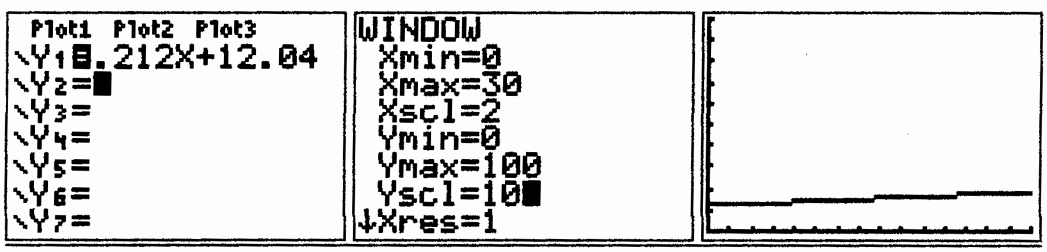
$y$ the percent of the labor force, must be between 0% and 100%.
The $y$-values tells us Ymin = 0 and Ymax = 100.

<u>Step 2:</u> Find Xscl and Yscl.

Xmax – Xmin = 30 – 0 = 30, which is divisible by 1 (30 ticks), 2 (15 ticks), or 5 (6 ticks). Choose Xscl = 2.
Ymax – Ymin = 100 – 0 = 100, which is divisible by 10 (10 ticks). Choose Yscl = 10.

<u>Step 3:</u> Graph.




<u>Step 4:</u>

The graph is increasing, so the percentage of the workforce which are 65 or older is increasing.
The values are between 10% and 20%, so most of the workforce is younger than 65.

**Example 3:** The equation $f(x) = 0.124x + 0.505$ tells the cost of a 30-second Super Bowl commercial, $f$, in millions of dollars, for $x$ the number of years since 1990. What types of numbers do not make sense for $x$? What types of numbers do not make sense for $y$? Graph $f(x) = 0.124x + 0.505$ in an appropriate window. Then find and interpret $f(20)$ and the year that the cost of one ad exceeds 3 million dollars.

<u>Step 1:</u> What values make sense? Which maximum and minimum values are easiest to calculate?

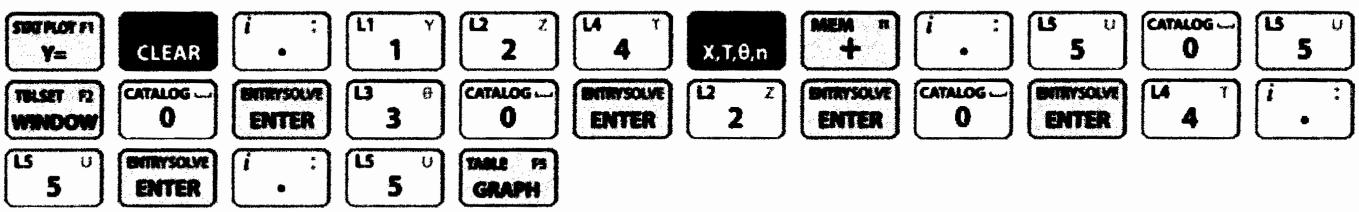
$x$ the number of years, is a whole number. If we consider 1990 and 2020, we get $x = 0$ and $x = 30$, giving Xmin = 0 and Xmax = 30.
$y$ in millions of dollars, must be positive.

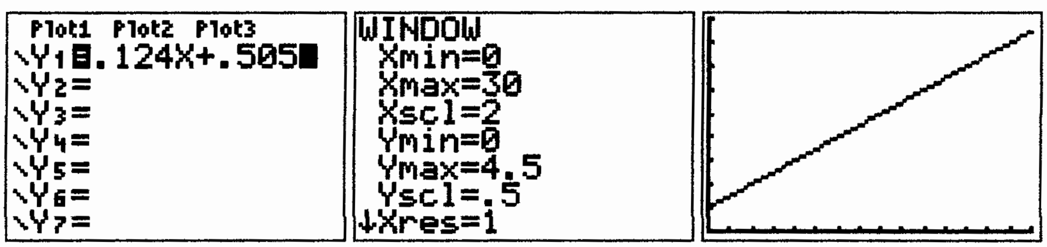
<u>Step 2:</u> Substitute to find ordered pairs on the graph to find remaining upper and lower values.

When $x = 0$, $f(0) = 0.124(0) + 0.505 = 0.505$ means that in 1990, one ad cost about half a million dollars.
Choose Ymin = 0 (since the origin is a good reference point).
When $x = 30$, $f(30) = 0.124(30) + 0.505 = 4.225$ means that in 2020, one ad may cost \$4.225 million.
Choose Ymax = 4.5

<u>Step 3:</u> Determine Xscl and Yscl.
Xmax – Xmin = 30, which is divisible by 2 (15 ticks), 3 (10 ticks) or 5 (6 ticks). Choose Xscl = 2 here.
Ymax – Ymin = 4.5, which is divisible by 0.5 (9 ticks). Choose Yscl = 0.5

<u>Step 4</u>: Graph.




<u>Step 5:</u> $f(20) = 0.124(20) + 0.505 = 2.985$ million dollars.
When $x$ is 20, the year is 1990 + 20 = 2010. The cost of an add in 2010 was \$2.985 million.

<u>Step 6:</u> To determine the year in which one ad exceeds 3 million dollars, replace $f(x)$ by 3.
$3 = 0.124x + 0.505$ gives $x \approx 20.12$

Because the Super Bowl happens only once per year, when $x = 20$ in 2010, the cost is below \$3 million. So 2011 is the first year that the cost exceeds \$3 million.

Practice

1) The amount *f(x)* spent on Research and Development for pharmaceuticals, in billions of dollars, spent each year $x$ is given by $f(x) = 2.602x - 5178$. The model is valid for $x$ beginning year 2000.
   a. Graph in an appropriate window.
   b. How much is spent in 2015?
   c. In what year does spending first exceed $70 billion?

2) The price of a useful gizmo $x$ in dollars, is related to the number of gizmos sold $y$ by the equation $y(x) = -1000x + 13000$.
   a. Graph in an appropriate window.
   b. How many gizmos are sold if the price is $8?
   c. What is the price of one gizmo if 3000 are sold?

3) The number of smartphones shipped per year, $y$, in millions, is related to the number of years since 2000, $x$, by the equation $y = 26.351x - 199.843$
   a. Graph in an appropriate window.
   b. How many smartphones were shipped in 2008?
   c. In what year does the number of smartphones shipped exceed 300 million?

4) The number of admissions to movie theatres $y$ in billions, for $x$ years after 2002, is given by $y(x) = -0.04x + 1.62$
   a. Graph in an appropriate window.
   b. How many admissions were purchased in 2004?
   c. In what year will the number of admissions first drop below one billion?

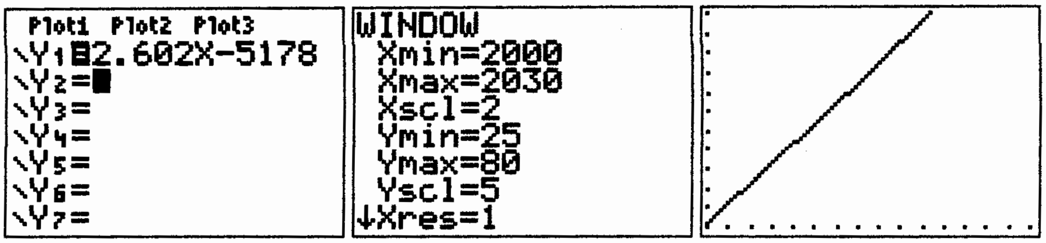
5) The cost of renting a minivan $C$, driven for $x$ miles is given by $C(x) = 0.3x + 42$
   a. Graph in an appropriate window.
   b. How many whole miles can be driven for $500?
   c. What is the cost of driving 2400 miles?

6) The fraction, expressed as a decimal, $y$ of females age 18 and under who smoke during year $x$, where $x$ is the number of years since 1960, is given by $y = -0.00391x + .36218$
   a. Graph in an appropriate window.
   b. What decimal, to the nearest thousandth, of females age 18 and under, smoked in 1970?
   c. Write your answer for part b as a percent.
   d. In what year did fewer than 18% of females age 18 and under smoke?

**TI-84+ GC 19 Choosing an Appropriate Window for Applications** page 5 Solutions

1) Note: $x$ is the actual year, (not the number of years since 2000).

a.

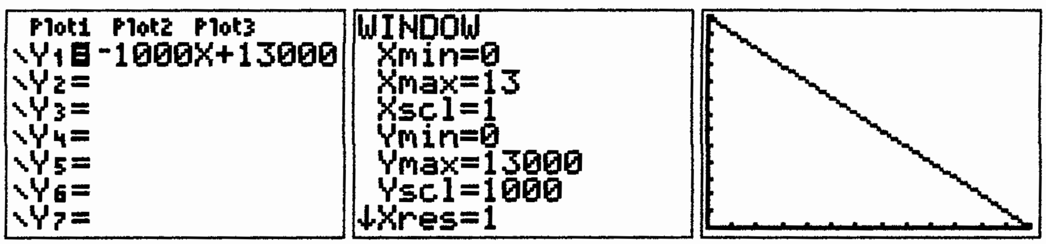



b. $f(2015) = 65.03$, $65.03 billion.

c. $70 = 2.602x - 5178$ gives $x \approx 2016.9$, 2017.

2) Note: find $x$ and $y$ intercepts.

a.

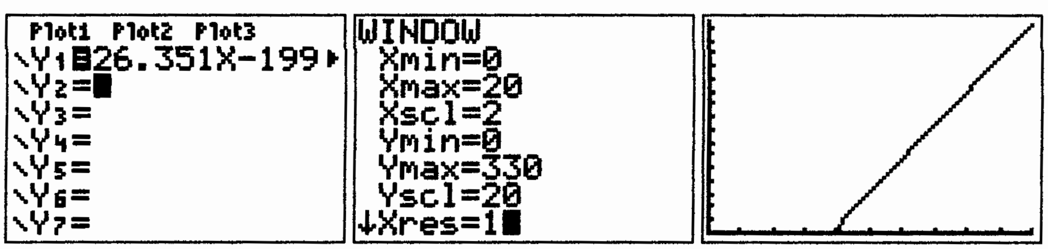



b. $y(8) = 5000$, 5000 gizmos

c. $3000 = -1000x + 13000$ gives $x = 10$, $10

3) Note: $y$ cannot be negative. For $x$, choose an ending year, like 2020.

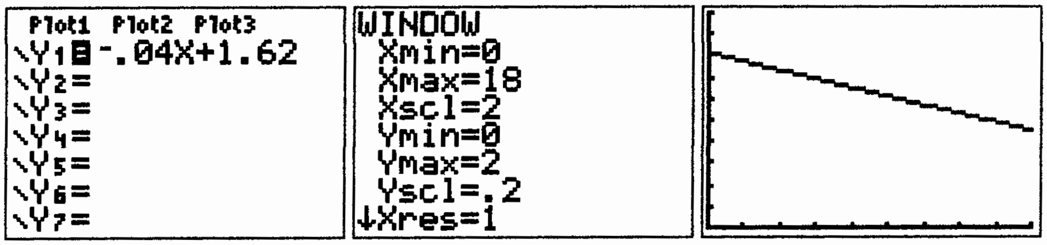
a.




b. $x = 2008 - 2000 = 8$, $26.351(8) - 199.843 = 10.965$, 10.965 million smartphones

c. $300 = 26.351x - 199.843$ gives $x \approx 18.97$, $2000 + 19 =$ 2019

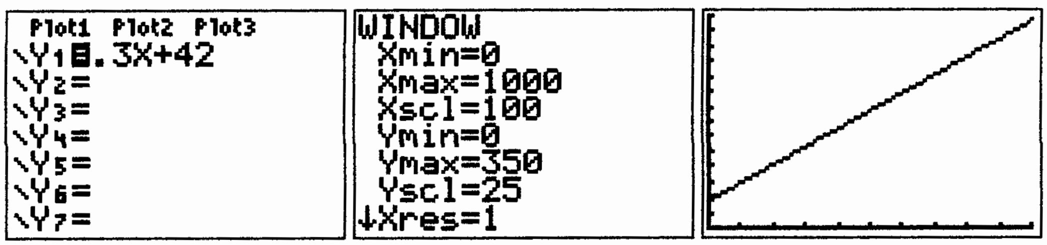
4) Note: include the origin as a point of reference

a.



b. $x = 2004 - 2002 = 2$ gives 1.54 billion admissions

c. $1 = -.04x + 1.62$ gives $x \approx 15.5$, 16+2002 = 2018

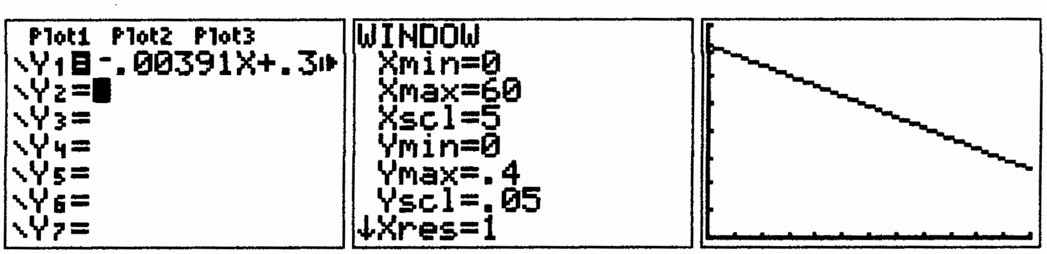
5) Note: include the origin as a point of reference.

a.



b. $500 = .3x + 42$ gives $x \approx 1526.7$, 1526 miles. (1527 miles costs more than $500).

c. $C(2400) = 762$, $762

6) Note; this function gives a percentage in its decimal form.

a.



b. $x = 1970 - 1960 = 10$, $y(10) = .32308 \approx .323$ 0.323

c. 32.3%

d. $.18 = -.00391x + 36218$ gives $x \approx 46.6$, $1960 + 47 = 2007$, 2007

Rev 8-8-13

Name ______________________________

Date ______________________________

## TI-84+ GC 20 Graphing More Than One Function

Objectives: Use MODE to graph two or more functions sequentially or simultaneously
Use and pronounce subscript notation
Turn graphs on and off without deleting them from the Y= menu
Use graphing styles in the Y= menu
Using TRACE and Zoom Integer (ZInteger) when multiple functions are graphed

Usually it's quicker to graph multiple functions simultaneously. In the MODE menu, change SEQUENTIAL to SIMUL:

MODE ∨ ∨ ∨ ∨ ∨ > ENTER Then quit: 2nd MODE


To graph up to ten functions in the same graphing window, use more functions in the Y= menu.

Use ∨ and ∧ to see all the functions in the Y= menu.

The Y= menu uses <u>subscript notation</u> to identify each function. A subscript is a number (or letter or word) which is written below the line next to the function name.

**Example 1:** The 1 in $y_1$ is a subscript, and $y_1$ is pronounced "y-sub-one".

Subscripts are often used in math for a list of similar objects.
The GC will graph up to ten functions, $y_1$, $y_2$, ... $y_9$, $y_0$.

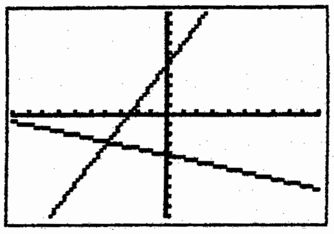
CAUTION: You can also scroll to the top where it says Plot1 Plot2 Plot3. These Plots should be off. If you press ENTER and turn on any of these three Plots, you'll get an error (see GC9).

(To turn off a Plot, use the ∧ and > to highlight the Plot that's on, and press ENTER).

**Example 2:** Graph $y = 2x + 5$ and $y = -\frac{1}{3}x - 4$ in a standard window. Use your knowledge of slope to identify which graph is which.

Input functions Y= // CLEAR 2 X,T,θ,n + 5 ENTER // CLEAR (−) X,T,θ,n ÷ 3 − 4 ENTER Graph in standard window. ZOOM 6 .

Example 2, continued.

Answer: 

The line going uphill (from left to right) has positive slope, so it must be $y = 2x + 5$, with slope 2.

The line going downhill (from left to right) has negative slope, so it's $y = -\frac{1}{3}x - 4$, with slope $-\frac{1}{3}$.

**Example 3:** Graph $f(x) = 2x^2 + 3x + 1$, $g(x) = 2x^2 + 3x - 1$, $h(x) = 2x^2 + 3x + 4$ and $k(x) = 2x^2 + 3x - 4$ in the same window.

Input the functions: [Y=]

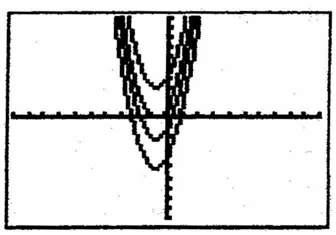
[CLEAR] [2] [X,T,Θ,n] [$x^2$] [+] [3] [X,T,Θ,n] [+] [1] [ENTER]

[CLEAR] [2] [X,T,Θ,n] [$x^2$] [+] [3] [X,T,Θ,n] [−] [1] [ENTER]

[CLEAR] [2] [X,T,Θ,n] [$x^2$] [+] [3] [X,T,Θ,n] [+] [4] [ENTER]

[CLEAR] [2] [X,T,Θ,n] [$x^2$] [+] [3] [X,T,Θ,n] [−] [4] [ENTER]

Graph in standard window: [ZOOM] [6]

Answer: 

When graphs are similar, there are three ways to help see which function is which:

Turn off all or most of the graphs by un-selecting the = next to that function in the Y= menu.

OR

Change MODE to SEQUENTIAL. The GC will graph $y_1$, then $y_2$, and so on, in order.

OR

Use a different graphing style for each graph by changing the symbol to the left of the function in the Y= menu.

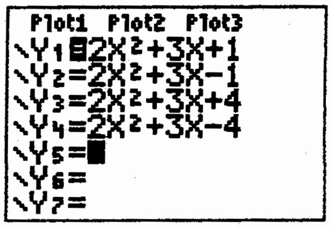
The next examples will demonstrate each of these.

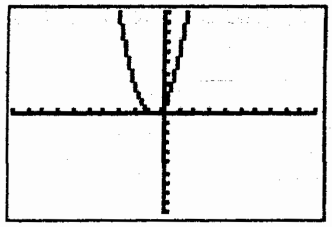
**Example 4:** Using the graphs from the previous example, turn off $y_2$, $y_3$, and $y_4$. Graph only $y_1$.

Open the Y= menu [Y=] Move to the = in front of $y_2$: [▼] [◄]

Unselect $y_2$: [ENTER].

Repeat for $y_3$ and $y_4$, then graph: [▼] [ENTER] [▼] [ENTER] [GRAPH].

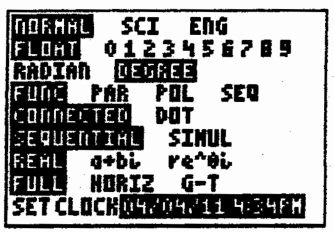
Answer: 

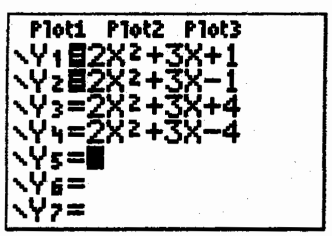


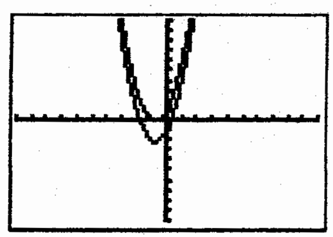
**Example 5:** Using the graphs from the previous examples, turn $y_2$ on, and graph $y_1$ and $y_2$ sequentially instead of simultaneously.

Change mode: [MODE] [▼] [▼] [▼] [▼] [▼] [ENTER] [2nd] [MODE]

Turn on $y_2$ and graph: [Y=] [▼] [◄] [ENTER] [GRAPH]

Answer: 





<u>To use different graphing styles:</u>

In the Y= menu, notice the diagonal line to the left of the equal sign. This is the "line" graphing style. If you move on top of this symbol, and press enter, you will change the "line" style to the "bold line" style. If you press enter again, you get "shade above" style. The GC goes through the list of available styles, then returns to the "line" style. The seven styles, in order, are:

- Line
- Bold line
- Shade above
- Shade below
- Bubble line
- Bubble invisible
- Dotted

When you clear a function from the Y= menu, it automatically returns to line style.

**Example 6:** Use the same functions from the previous examples. Turn $y_3$ and $y_4$ on, then graph $y_1$ in line style, $y_2$ in bold line style, $y_3$ in bubble line style, and $y_4$ in dotted line style.

Turn on functions: [Y=] move to $y_3$: [▼] [▼] move to =: [◄] select: [ENTER]

move to $y_4$: [▼] select: [ENTER]

Move back to top and left of $y_2$: [▲] [▲] [◄]

Change line style to bold line style [ENTER]

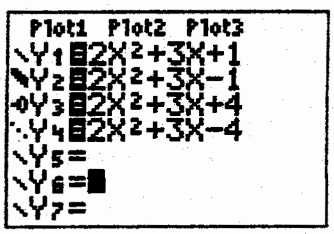
Move $y_3$ [▼]

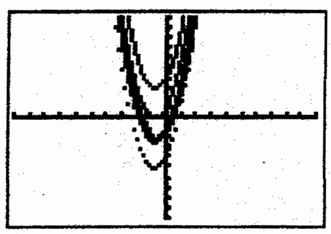
Change line style to bubble line style: [ENTER] [ENTER] [ENTER] [ENTER]

Move to $y_4$ [▼]

Change line style to dotted line style: [ENTER] [ENTER] [ENTER] [ENTER] [ENTER] [ENTER]

Graph: [GRAPH]

Answer: 



When using Zoom Integer and Trace with multiple functions, read the information on the screen carefully. The graphs may be hard to see, but it won't matter.

Use [▼] to move down the list of functions in the Y= menu (or [▲] to go up the list).

Use [►] or [◄] to move to new x-values.

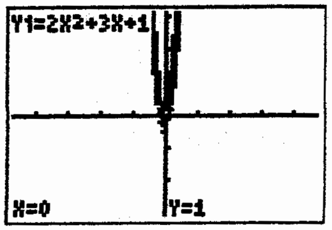
**Example 7:** Use Zoom Integer (ZInteger) and TRACE to complete the tables for the functions in the previous examples.

| X | $y_1$ | $y_2$ | $y_3$ | $y_4$ |
|---|---|---|---|---|
| -1 | | | | |
| 0 | | | | |
| 2 | | | | |

Example 7, continued.

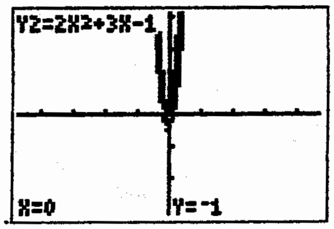
Zoom Integer [ZOOM] [8]. Move to origin if necessary and press [ENTER]. Trace [TRACE].

At the top of the screen is the function and at the bottom of the screen are the x- and y-values.



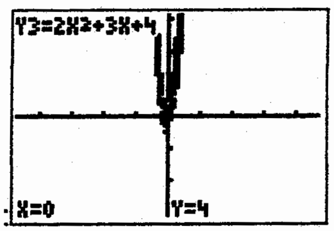

This screen shows x=0 gives $y_1$ = 1.

Press [▼] to move to $y_2$.



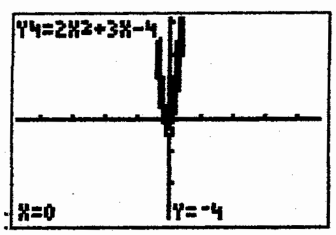

This screen shows x=0 and $y_2$=-1

Press [▼] to move to $y_3$:




This screen shows x=0 and $y_3$ = 4

Press [▼] to move to $y_4$:




This screen shows x=0 and $y_4$=-4

Press [◄] to move to x=-1, then [▲] three times to move back up the list of functions, noting the values.

Press [►] [►] [►] to move to x=2 then press [▼] to move back down the list of functions and note the values.

Answer:

| x | $y_1$ | $y_2$ | $y_3$ | $y_4$ |
|---|---|---|---|---|
| -1 | 0 | -2 | 3 | -5 |
| 0 | 1 | -1 | 4 | -4 |
| 2 | 15 | 13 | 18 | 10 |

Practice:

1) Graph $y_1 = 3x - 7$ and $y_2 = -x^2 + 4$ simultaneously, both with line style, in the standard window.

2) Graph $y_1 = 3x - 7$ and $y_2 = -x^2 + 4$ sequentially, both with line style, in the standard window.

3) Graph $y_1 = 3x - 7$ and $y_2 = -x^2 + 4$, then turn off the graph for $y_1$ and graph only $y_2$.

4) Turn off the graph for $y_2$ in the previous question and graph only $y_1 = 3x - 7$ with each of the seven styles. Which style do you like best?

5) Complete the table for $y_1 = 3x - 7$ and $y_2 = -x^2 + 4$.

| X | $y_1$ | $y_2$ |
|---|---|---|
| -2 | | |
| -1 | | |
| 0 | | |
| 1 | | |
| 2 | | |

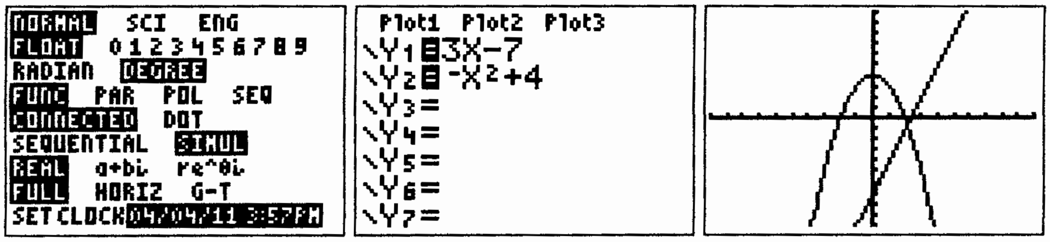
6) Complete the table for $y_1 = 3x - 7$, $y_2 = -x^2 + 4$, $y_3 = x^3$

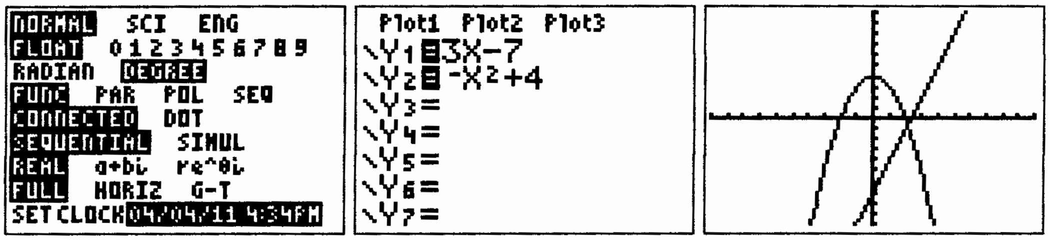
| X | $y_1$ | $y_2$ | $y_3$ |
|---|---|---|---|
| -2 | | | |
| -1 | | | |
| 0 | | | |
| 1 | | | |
| 2 | | | |

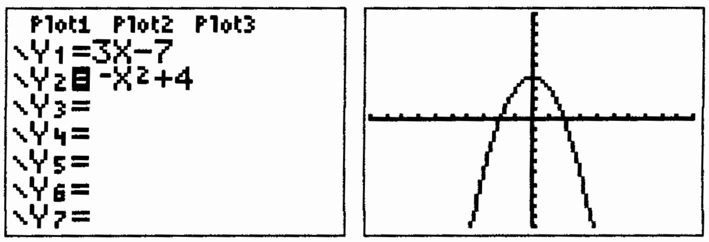
7) Complete the table for $y_1 = 3x - 7$, $y_2 = -x^2 + 4$, $y_3 = x^3$, and $y_4 = -\frac{1}{6}x + 2$

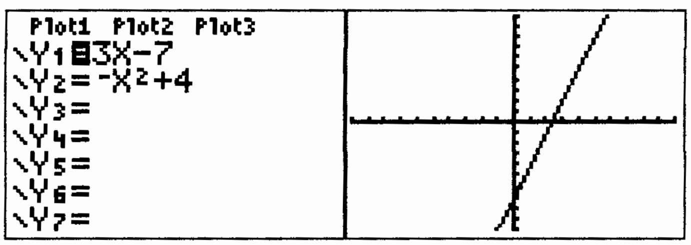
| X | $y_1$ | $y_2$ | $y_3$ | $y_4$ |
|---|---|---|---|---|
| -1 | | | | |
| 0 | | | | |
| 2 | | | | |
| 7 | | | | |

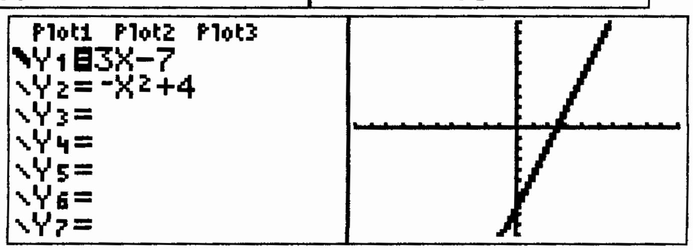
# TI-84+ GC 20 Graphing More Than One Function page 7, solutions

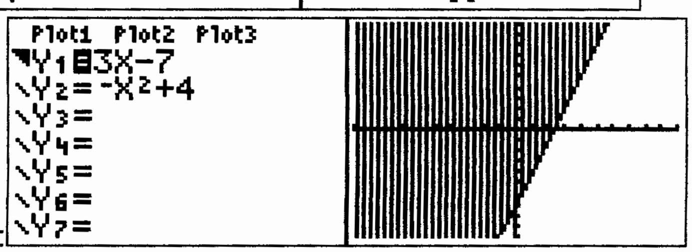
1) 

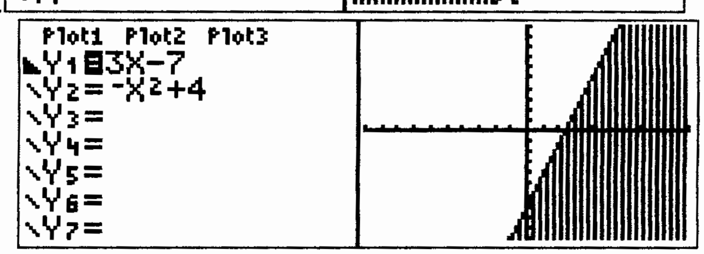

2) 


3) 


4) line style 

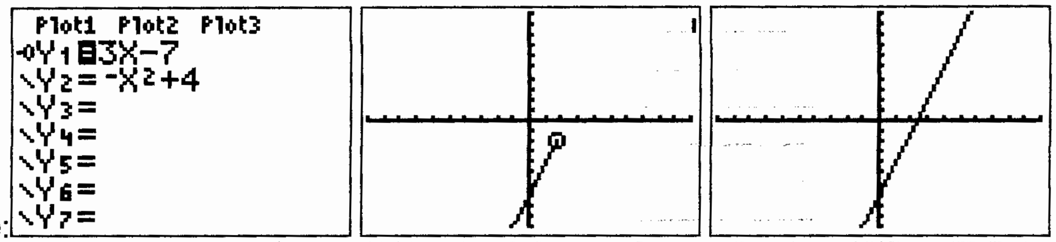

bold line style: 


shade above style: 

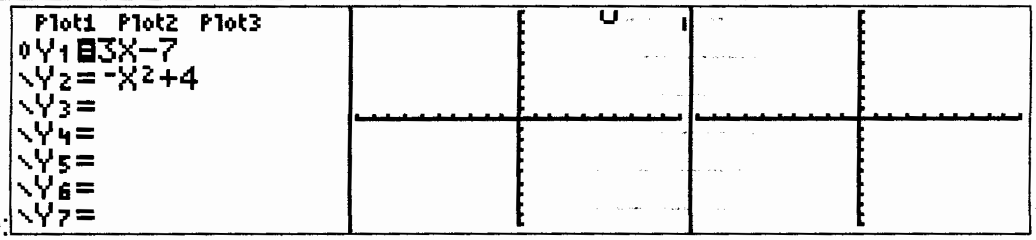

shade below style: 

4 continued.

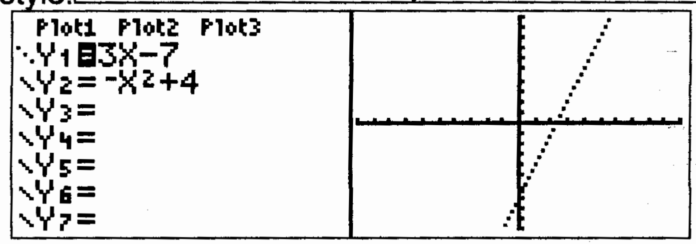
bubble line style:



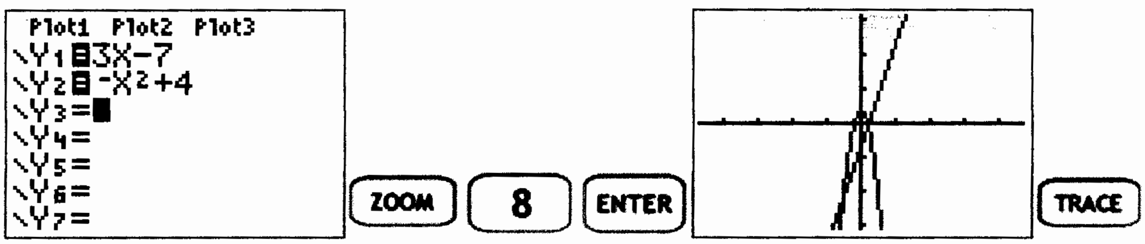

bubble invisible style:




dotted line style:




5)




| x | $y_1$ | $y_2$ |
|---|---|---|
| -2 | -13 | 0 |
| -1 | -10 | 3 |
| 0 | -7 | 4 |
| 1 | -4 | 3 |
| 2 | -1 | 0 |

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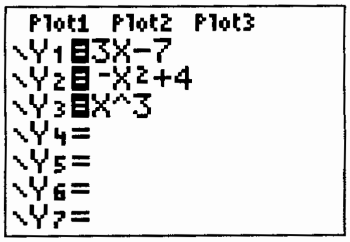
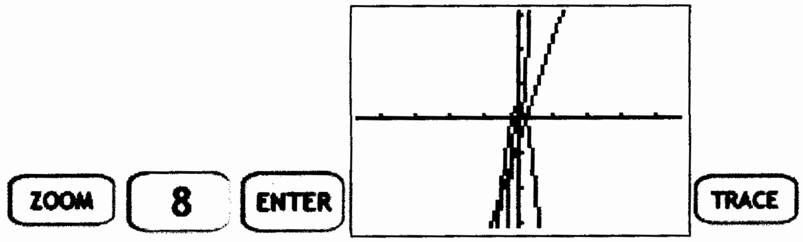
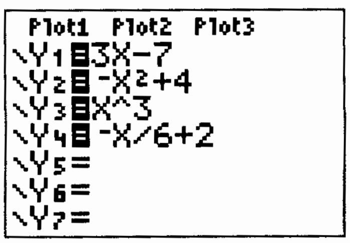
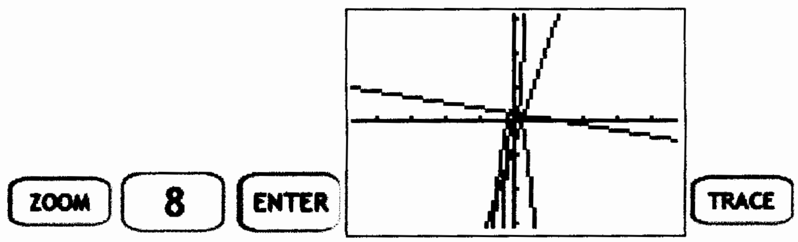
## TI-84+ GC 20 Graphing More Than One Function page 9, solutions

6) 


| x | $y_1$ | $y_2$ | $y_3$ |
|---|---|---|---|
| -2 | -13 | 0 | -8 |
| -1 | -10 | 3 | -1 |
| 0 | -7 | 4 | 0 |
| 1 | -4 | 3 | 1 |
| 2 | -1 | 0 | 8 |

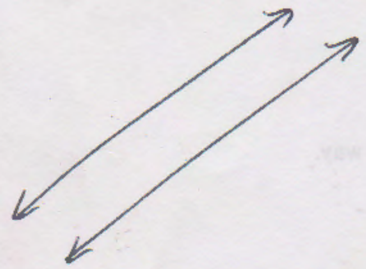
7) 

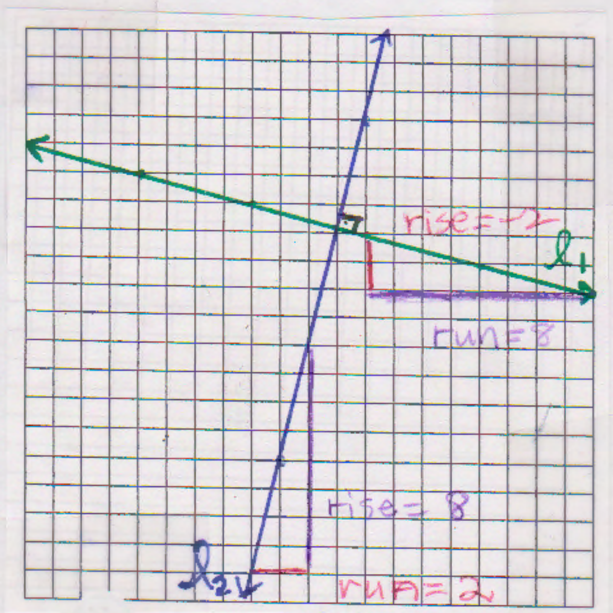

| x | $y_1$ | $y_2$ | $y_3$ | $y_4$ |
|---|---|---|---|---|
| -1 | -10 | 3 | -1 | $2.1\overline{6}$ |
| 0 | -7 | 4 | 0 | 2 |
| 2 | -1 | 0 | 8 | $1.\overline{6}$ |
| 7 | 14 | -45 | 343 | $8.\overline{3}$ |

M70

**Parallel Lines**:
- never intersect
- go the same direction
- at least two lines or it doesn't make sense
- have the same slope
  m= same number.



**Perpendicular Lines**
- intersect in a right angle or a 90° angle
- at least two lines or it doesn't make sense
- slopes must be both
  1) opposites
  2) reciprocals




Example:

slope $l_1 = \frac{-2}{8} = \frac{-1}{4}$

slope $l_2 = \frac{8}{+2} = 4$